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Developing the D-MLR framework: Mathematics learning readiness as goal- and pathway-conditioned resource adequacy

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Abstract

Mathematics learning readiness cannot be equated with prior knowledge or the ability to produce correct answers, as neither necessarily indicates whether learners possess adequate resources to address specific mathematical goals and task demands. This study aims to develop the *Dynamic Mathematics Learning Readiness* (D-MLR) framework, which explains readiness in relation to competence goals, solution pathways, and permissible assistance conditions. Framework development involves targeted literature synthesis, comparison of theoretical perspectives, and conceptual stress testing across different goals, solution pathways, and assistance conditions. The resulting framework integrates cognitive, motivational, self-regulatory, and digital resources within a learner–task–time relationship, while considering contextual access and learning opportunities. D-MLR distinguishes readiness to learn from readiness to perform and establishes a bounded compensation principle for external assistance, preventing assisted performance from being misinterpreted as evidence of independent competence. The framework provides an analytical foundation for instructional decisions that respond to learners' diverse circumstances and supports efforts to promote equitable access to mathematics learning through subsequent empirical evaluation.

Keywords: Mathematics learning readiness; Resource adequacy; Mathematical solution pathways; Educational access inequality; Independent mathematical competence.

Abstrak

Kesiapan belajar matematika tidak dapat disamakan dengan penguasaan pengetahuan awal atau keberhasilan memperoleh jawaban yang benar karena keduanya belum tentu menunjukkan kecukupan sumber daya peserta didik dalam menghadapi tujuan dan tuntutan aktivitas matematika tertentu. Penelitian ini bertujuan mengembangkan *Dynamic Mathematics Learning Readiness* (D-MLR), yaitu kerangka yang menjelaskan kesiapan belajar berdasarkan tujuan kompetensi, jalur penyelesaian, dan kondisi bantuan yang diperbolehkan. Pengembangan kerangka dilakukan melalui sintesis literatur terarah, perbandingan perspektif teoretis, dan pengujian konsistensi konseptual terhadap variasi tujuan, jalur penyelesaian, serta kondisi bantuan. Hasil pengembangan menghasilkan kerangka yang mengintegrasikan sumber daya kognitif, motivasional, regulasi diri, dan digital dalam hubungan peserta didik–tugas–waktu, dengan mempertimbangkan konteks akses dan kesempatan belajar. D-MLR membedakan kesiapan untuk belajar dan kesiapan untuk menyelesaikan tugas, serta menetapkan prinsip kompensasi terbatas bagi bantuan eksternal agar kinerja berbantuan tidak disalahartikan sebagai bukti kompetensi independen. Kerangka ini memberikan landasan analitis bagi pengambilan keputusan pembelajaran yang responsif terhadap keragaman kondisi peserta didik dan mendukung upaya pemerataan kesempatan belajar matematika melalui pengujian empiris lebih lanjut.

Kata Kunci: Kesiapan belajar matematika; Kecukupan sumber daya; Jalur penyelesaian matematika; Kesenjangan akses pendidikan; Kompetensi matematika independen.

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Introduction

Mathematics learning readiness is often assessed in terms of prior knowledge, although initial ability alone does not explain how learners respond to different task demands. Using multisite longitudinal data, Watts et al. (2014) demonstrated that mathematical ability at 54 months predicted mathematics achievement through age 15, even after accounting for reading ability, cognitive ability, and family and child characteristics. Geary (2011) also followed 177 students from the beginning of Grade 1 through Grade 5 and found that early quantitative abilities, intelligence, processing speed, and certain components of working memory were associated with initial achievement levels or subsequent growth in mathematics achievement. These findings underscore the importance of initial resources but do not establish whether those resources are adequate for particular goals and solution pathways within specific learning activities.

Mathematical knowledge is not the only factor influencing learners' engagement in learning activities. Affective factors are also associated with how learners respond to mathematical demands. In a meta-analysis combining 747 effect sizes from studies conducted between 1992 and 2018, Barroso et al. (2021) found a significant negative association between mathematics anxiety and mathematics achievement (r=-.28). Meanwhile, through a systematic review and meta-analysis of longitudinal studies, Ruiz et al. (2024) examined domain-specific numeracy skills at the beginning of formal schooling as predictors of subsequent mathematics achievement. They found that the available evidence was insufficient to establish any single skill as the strongest predictor, partly because of variation across studies and inconsistencies in measurement. This suggests that readiness should be understood through a combination of relevant resources rather than a single indicator or an initial ability score alone.

Differences in readiness also become apparent when learners encounter tasks that allow more than one solution strategy. Elia et al. (2009) examined 152 high-achieving fourth-grade students in the Netherlands using three non-routine problems. Students who were able to switch strategies across tasks tended to perform better than those who consistently used the same strategy; however, switching strategies within a single task did not automatically improve answer accuracy. In the context of teacher education, Schulz (2024) examined the procedural fluency and strategic competence of prospective primary school teachers in operations involving natural and rational numbers. The study positioned the flexible use of mental models, strategies, and procedures as components of mathematical competence. Together, these studies demonstrate that correct answers alone are insufficient to characterize the strategies and reasoning learners possess, particularly when learning objectives emphasize a specific method.

Research on instructional assistance and technology also indicates that success during task completion must be distinguished from competence development. Through an experiment involving multi-step mathematical problem-solving, Chen et al. (2023) found that worked examples produced better outcomes than problem solving without examples on retention and transfer tests, while also reducing cognitive load and learning challenges. In contrast, Rakes et al. (2020), in a systematic review and meta-analysis of educational technology in mathematics, found that the effects on mathematics achievement and orientation were relatively small; after adjustment for publication bias, the effects declined to near zero. Moreover, 82% of the studies analyzed focused exclusively on procedural understanding. These findings indicate that assistance and technology should not be evaluated solely in terms of their availability but must be considered in relation to task demands and the competencies intended to be developed.

In response to these issues, this study aims to develop the *Dynamic Mathematics Learning Readiness* (D-MLR) framework, which conceptualizes readiness as the prospective adequacy of learners' resources in relation to competence goals, task demands, solution pathways, context, and permissible assistance. The framework integrates cognitive, motivational, self-regulatory, and digital resources while distinguishing readiness to learn from readiness to perform a task. D-MLR also explains the limits of external assistance as a compensatory resource and distinguishes immediate performance from the development of independent competence. Developing this framework is important because instructional decisions cannot rely solely on general scores or correct answers. An analytical basis is needed to determine which activities learners are ready to engage in, what forms of support are appropriate, and what evidence can be used to assess the competencies that have genuinely developed.

Method

This study adopts a conceptual research design through literature synthesis and theoretical framework development (*conceptual synthesis and framework development*). This approach aims to integrate various theoretical perspectives to construct a new explanatory framework rather than test empirical relationships or conduct a systematic review (Jaakkola, 2020; MacInnis, 2011). The data sources consist of scholarly literature relevant to learning readiness, mathematics learning, self-regulation, cognitive diagnosis, instructional assistance, and competence development. The literature was purposively selected based on its conceptual relevance, contribution to the research problem, bibliographic clarity, and availability of source information. Data collection involved a targeted literature search for publications addressing goal-oriented decision-making, *knowledge space theory*, *self-regulated learning*, instructional assistance, and mathematical competence assessment.

This procedure follows the principle that conceptual framework development should explicitly explain source selection, the role of theoretical perspectives, and the logic underlying their integration (Jabareen, 2009; Jaakkola, 2020).

The analysis involved comparative conceptual synthesis and logical consistency testing (*conceptual stress testing*). First, the selected literature was compared in terms of its explanatory focus, unit of analysis, conceptualization of learner resources, treatment of external assistance, and basis for deriving evidence of competence. Second, relevant concepts were integrated to formulate relationships between cognitive, motivational, self-regulatory, and digital resources and competence goals, task demands, and alternative solution pathways. Third, the relationships among these components were examined by comparing different mathematical goals, solution pathways, and assistance conditions. For example, solving quadratic equations was used to examine whether correct answers obtained through different methods could represent different target competencies. Counterexamples were also used to identify conditions under which assistance enables task completion without independent mastery or, alternatively, supports the development of new strategies. The analytical outcomes were formulated as definitions, decision rules, and conceptual propositions within the D-MLR framework. Logical consistency was examined deductively, while empirical validity remains subject to further investigation (Corley & Gioia, 2011; MacInnis, 2011; Whetten, 1989).

Results and Discussion

The problem of mathematics learning readiness: Between prior mastery and learning opportunities

Mathematics learning readiness is still frequently understood in terms of mastery of prerequisite knowledge and learners' ability to complete tasks. This understanding creates a problem when readiness is equated with the ability to perform the very material that constitutes the intended learning outcome. Learners who are unable to independently execute a mathematical procedure are not necessarily unprepared to learn it. Conversely, learners who successfully obtain correct answers do not necessarily possess the reasoning underlying the solutions. Thus, prior mastery and successful task completion cannot serve as the sole indicators of learning readiness. This issue becomes increasingly complex because mathematics learning is also associated with psychological conditions, environmental support, educational resources, and opportunities to engage in learning activities. Readiness therefore needs to be understood as a broader issue than merely an individual's initial ability.

Table 1. Problems in understanding mathematics learning readiness

Problem	Research findings	Problem identified
Reducing readiness to prior knowledge mastery	The level of mathematical knowledge required to enter university-level learning cannot be adequately explained by the amount of knowledge already mastered. The structure of knowledge must be aligned with the demands of subsequent material (Rach & Ufer, 2020).	Mastery of previous material does not automatically indicate readiness to develop new competencies.
Neglecting learners' psychological conditions	Cognitive ability, motivation, perceived competence, and anxiety are associated with mathematics achievement (Abín et al., 2020). Mathematics anxiety also shows a negative association with achievement, with a correlation of (r=-.28) (Barroso et al., 2021).	Readiness assessments focused exclusively on cognitive ability overlook the emotional and motivational conditions accompanying the learning process.
Equating the intensity of support with its quality	More intensive parental involvement was found to be associated with lower mathematics achievement in the context examined by Kusaeri et al. (2018). Jay et al. (2018) also showed that school-centered approaches to parental involvement can limit parents' understanding of how to support their children's learning at home.	The presence of family support does not automatically indicate that the support is appropriate to learners' needs.
Neglecting inequalities in learning resources	Educational resources at home and the socioeconomic composition of schools are associated with mathematics achievement. Self-confidence also emerges as an important predictor (Yıldırım, 2019).	Individual-focused readiness assessments overlook differences in social conditions and resources that shape learning opportunities.
Equating access to technology with learning opportunities	Computer use in mathematics instruction is indirectly associated with achievement through the learning opportunities provided by teachers (Zhang & Wang, 2020).	The availability of technology does not automatically ensure that learners have meaningful learning opportunities.
Equating successful task completion with independent competence	Worked examples can support knowledge retention and transfer while reducing cognitive load in mathematics learning (Chen et al., 2023).	Task outcomes achieved with assistance do not necessarily demonstrate independent ability, although assistance does not always hinder learning.

Source: Compiled based on Rach and Ufer (2020), Abín et al. (2020), Barroso et al. (2021), Kusaeri et al. (2018), Yıldırım (2019), Jay et al. (2018), Zhang and Wang (2020), and Chen et al. (2023).

The findings in Table 1 demonstrate that the problems surrounding mathematics learning readiness lie not only in limitations in initial ability but also in how readiness is identified and interpreted. The six problems indicate a tendency to assess readiness

through indicators that are detached from the demands of the learning activities learners are expected to undertake. Prior knowledge, psychological conditions, family support, socioeconomic resources, access to technology, and instructional assistance are often discussed as factors associated with mathematics achievement. However, these factors do not necessarily explain whether learners possess adequate resources to engage in particular activities, complete tasks, or develop competence independently. Therefore, the central issue is not simply the addition of more indicators but the need to establish the relationships between available resources, learning demands, and expected forms of engagement.

The first problem concerns the tendency to equate readiness with the amount of mathematical knowledge already mastered. Through a reanalysis of data from a previous study involving first-year university students, Rach and Ufer (2020) demonstrated that success in university mathematics courses cannot be adequately explained by the general assumption that more prior knowledge is always better. Relevant knowledge must also be structured and understood at a level appropriate to the demands of university mathematics. This finding indicates that prior knowledge cannot be assessed solely in terms of the quantity of material previously learned. Its relevance to subsequent learning activities is far more consequential. The implication is that learners who have not mastered all prerequisite material are not automatically unprepared to learn. Conversely, mastery of several routine procedures does not guarantee readiness to engage in activities requiring conceptual understanding or different forms of reasoning.

The second problem arises when readiness is reduced to cognitive ability while neglecting the psychological conditions accompanying learning. In a study involving 2,365 secondary school students in Spain, Abín et al. (2020) examined the relationships between intellectual ability, perceived competence, perceived usefulness of mathematics, intrinsic interest, anxiety, and motivational variables and mathematics achievement. Meanwhile, a meta-analysis of 223 studies by Barroso et al. (2021) found a negative relationship between mathematics anxiety and achievement, with a mean correlation of −0.28. These studies demonstrate that mathematics achievement cannot be separated from learners' cognitive and psychological conditions. Nevertheless, these relationships do not establish that psychological conditions directly determine readiness. Rather, the findings highlight the limitations of assessments that rely exclusively on content mastery without considering the conditions accompanying learners' engagement in learning.

The third problem concerns the assumption that the more intensive the family support, the better the learner's learning conditions. In a study involving 2,867 Indonesian senior high school students, Kusaeri et al. (2018) found that mothers' educational attainment contributed more strongly to mathematics achievement than fathers'

educational attainment. Maternal involvement also mediated the relationship between mothers' education and mathematics achievement. However, more intensive parental involvement was found to be associated with lower mathematics achievement in the context examined. This finding should not be interpreted as evidence that parental involvement is detrimental to learning. Instead, it indicates that the intensity of support is not necessarily equivalent to its quality or appropriateness. Jay et al. (2018) also demonstrated that approaches to parental involvement that are overly centered on school demands can limit parents' understanding of how to support mathematics learning at home. Accordingly, family support needs to be examined in terms of its form, function, and alignment with children's learning needs.

The fourth problem is the tendency to position readiness as an individual characteristic without accounting for inequalities in resources and learning opportunities. Yıldırım (2019), using data from TIMSS 2011 in Turkey, demonstrated that educational resources at home and the socioeconomic composition of schools were positively associated with mathematics achievement. Self-confidence also emerged as an important predictor in the model examined. These findings indicate that mathematics achievement is associated not only with what learners possess individually but also with the environments in which they learn. Learners may face the same learning demands under different resource conditions. Differences in access to learning materials, supportive environments, and educational facilities may create unequal opportunities to develop mathematical abilities. Therefore, low levels of prior mastery should not immediately be interpreted as individual deficiencies without considering the conditions that constrain learners' opportunities to learn.

A similar problem can be observed in the use of educational technology. The availability of digital devices is often treated as an indication that learners have adequate support for learning mathematics. However, access to technology is not equivalent to opportunities to obtain meaningful learning experiences. Through an analysis of TIMSS data, Zhang and Wang (2020) showed that computer use in mathematics instruction was indirectly associated with achievement through opportunities to learn content provided by teachers. This finding shifts attention from the mere use of devices to the quality of learning activities enabled by technology. In other words, technology has educational relevance only when its use is connected to appropriate content, activities, and instructional support. Therefore, readiness assessments should not simply ask whether learners have access to devices or can operate them. More importantly, they should examine whether these resources can be used to engage in and carry out the intended mathematical activities.

The sixth problem concerns interpreting successful task completion as evidence that learners have independently acquired competence. In mathematics learning, learners may obtain correct answers through worked examples, teacher guidance, calculators,

or digital assistance. However, such success does not necessarily reveal who actually performed the central reasoning involved in solving the task. Nevertheless, external assistance cannot automatically be considered detrimental to learning quality. Through an experiment on worked examples in multi-step mathematical problems, Chen et al. (2023) found that worked examples supported knowledge retention and transfer while reducing cognitive load in the context of their study. This finding indicates that assistance can serve as a productive learning resource when designed to support the process of understanding and learning solution steps. The issue lies in the function of the assistance: whether it creates opportunities for learners to develop their reasoning or instead replaces the reasoning they are expected to perform independently.

Overall, these six problems indicate that understanding mathematics learning readiness continues to face conceptual challenges in connecting prior ability, learner conditions, the learning environment, and the demands of learning activities. Prior knowledge is indeed important, but its relevance depends on the competence to be developed. Family and technological support can expand learning opportunities, but their presence does not automatically guarantee productive engagement. Similarly, successful task completion does not always provide evidence of independently acquired competence, while the use of assistance does not necessarily mean that learners are unable to learn. Thus, readiness is inadequately understood when treated as a general score or a fixed attribute of individuals. It needs to be interpreted in relation to learners' available resources, the demands of mathematical activities, the learning opportunities provided, and the forms of assistance permitted. This need provides the basis for the subsequent discussion of the relationship between individual resources, learning demands, and social context.

Mathematics learning readiness in the relationship between individual resources, learning demands, and social context

Mathematics learning readiness needs to be understood as a relationship between the resources learners possess and the mathematical activities they are expected to undertake, rather than as a standalone individual characteristic. Simon and Tzur (2004) explain that hypothetical learning trajectories connect mathematics learning goals, instructional tasks, and conjectures about the processes through which learners develop understanding. This perspective indicates that readiness cannot be separated from the goals and characteristics of learning activities. Verschaffel et al. (2009) also conceptualize adaptive expertise in mathematics as the ability to use strategies appropriate to individual, task, and contextual characteristics. These two perspectives emphasize that the same resources may have different levels of relevance when competence goals, task demands, and learning conditions change. Accordingly,

readiness assessment must consider not only what learners already possess but also the mathematical activities for which those resources are required.

Table 2. Components of mathematics learning readiness and their contributions to learning

Component	**Elements covered**	**Contribution to learning readiness**	**Interpretive limitations**
Cognitive resources	Conceptual and procedural knowledge, representations, and mathematical strategies	Provide prior knowledge and strategies for understanding and undertaking mathematical activities	The amount of knowledge does not automatically indicate its suitability for particular task demands
Motivational resources	Self-efficacy, success expectations, task value, and willingness to engage	Support activity choices, engagement, effort, and persistence in learning	Motivation and self-confidence do not demonstrate mastery of mathematical prerequisites
Self-regulatory resources	Planning, monitoring, strategy selection, evaluation, and adjustment of the learning process	Help learners manage their learning processes and respond to difficulties during task completion	Knowledge of regulatory strategies does not guarantee their effective use in every task
Digital resources	Device use, information search and evaluation, and technology utilization	Support information access, the use of representations, and the completion of technology-based activities	Device ownership and general digital skills do not demonstrate mathematical reasoning ability
Learning demands	Competence goals, task characteristics, prerequisites, solution pathways, and forms of assistance	Determine the types and levels of resource adequacy required to engage in or complete activities	Success on one task does not demonstrate readiness for different goals or solution pathways
Social and educational context	Support from teachers, families, and peers; facilities; access; and learning opportunities	Influence the availability, development, and utilization of individual resources	Inequalities in learning opportunities cannot be directly interpreted as deficiencies in individual ability

Source: Conceptual synthesis based on Simon and Tzur (2004), Verschaffel et al. (2009), Kalyuga et al. (2003), Pajares and Miller (1994), Eccles and Wigfield (2020), Efklides (2011), Panadero (2017), Winne and Hadwin (1998), Koedinger et al. (2012), Vuorikari et al. (2022), Alreshidi (2023), Yang et al. (2024), Trias et al. (2021), and Poçan et al. (2023).

Table 2 shows that mathematics learning readiness is constituted through the relationship between four families of individual resources, learning demands, and the social and educational context. Each resource serves a different function, but they do not operate independently. Cognitive resources provide knowledge and strategies,

motivational resources support engagement, self-regulatory resources guide the management of learning processes, and digital resources enable the use of technology according to the requirements of an activity. The relevance of all these resources is determined by competence goals and the characteristics of the tasks learners encounter. At the same time, social support, facilities, and learning opportunities influence the extent to which these resources can be developed and utilized. For example, Yang et al. (2024) showed that mathematics self-efficacy was associated with achievement across several educational levels while also identifying the influence of school context. Thus, readiness is determined not merely by the availability of resources but by their alignment with learning demands and the conditions that enable their utilization.

Cognitive resources provide the foundation for understanding the relationship between prior knowledge and the mathematical demands of an activity. The adequacy of these resources cannot be determined solely by the amount of material already mastered but must also be assessed in terms of the relevance of knowledge and strategies to learning objectives. Through a quasi-experimental study involving 73 students, Alreshidi (2023) found that strengthening topic-specific prior mathematical knowledge produced better procedural outcomes on both immediate and delayed tests. For conceptual knowledge, an advantage for the experimental group was found on the delayed test but not on the immediate test. These findings indicate that relevant prior knowledge can support the development of new knowledge, although its effects do not necessarily emerge uniformly across different types of knowledge and measurement points. Kalyuga et al. (2003), through their examination of the *expertise reversal effect*, also demonstrated that the effectiveness of instructional assistance depends on learners' initial levels of expertise. Meanwhile, Koedinger et al. (2012) connected types of knowledge, learning processes, and instructional events through the *Knowledge-Learning-Instruction* framework. Together, these perspectives emphasize that cognitive readiness must be assessed in terms of the alignment between prior knowledge, the processes required, and the characteristics of instruction.

Motivational resources determine learners' willingness to initiate, sustain, and direct their engagement in mathematics learning. Through path analysis involving 350 university students, Pajares and Miller (1994) found that mathematics self-efficacy was an important predictor of mathematical problem-solving performance. This finding indicates that beliefs about one's own capabilities are associated with how individuals approach mathematical tasks, but they cannot be equated with knowledge mastery. More recent evidence is provided by Yang et al. (2024), who analyzed 2019 NAEP data using multilevel modeling involving Grade 4, 8, and 12 students in the United States. Mathematics self-efficacy was found to be a significant predictor of mathematics achievement across all three grade levels, while school characteristics were also

associated with variations in achievement. Through their *situated expectancy-value theory*, Eccles and Wigfield (2020) explain that success expectations and task values are associated with choices, performance, and engagement, while also being shaped by developmental experiences and sociocultural environments. Overall, these findings position motivation as a resource associated with engagement and achievement, rather than as a substitute for mathematical prerequisites.

Self-regulatory resources concern learners' ability to plan, monitor, select strategies, evaluate, and adjust their learning processes. Through the *Metacognitive and Affective Model of Self-Regulated Learning* (MASRL), Efklides (2011) distinguishes between characteristics at the individual level (*Person level*) and processes emerging from interactions between individuals and tasks (*Task* × *Person level*). Through a review of six models of self-regulated learning, Panadero (2017) demonstrated that these processes encompass cognitive, metacognitive, behavioral, motivational, and emotional dimensions. Winne and Hadwin (1998) explain learning management through the COPES framework, comprising *Conditions, Operations, Products, Evaluations,* and *Standards*. Empirical support for the importance of self-regulation in mathematics is provided by Trias et al. (2021). In a quasi-experimental study involving 69 sixth-grade students with low mathematics achievement, cognitive, metacognitive, and volitional strategy interventions were implemented over 16 sessions. All three intervention groups demonstrated better problem-solving outcomes than the control group following the intervention, while metacognitive and volitional strategies showed stronger positive effects that persisted at the two-month follow-up. These findings indicate that self-regulatory strategies can be developed through instruction and are associated with improvements in problem-solving. Therefore, D-MLR needs to distinguish between available self-regulatory resources and the regulatory processes actually enacted when learners undertake tasks.

Digital resources encompass the ability to use and evaluate technologies relevant to learning activities. Through the *DigComp 2.2* framework, Vuorikari et al. (2022) identified five areas of digital competence: information and data literacy, communication and collaboration, digital content creation, safety, and problem-solving. This framework provides a general classification of digital competence rather than direct evidence of mathematics readiness or mathematical reasoning ability. Through a mixed-methods study involving 73 seventh-grade students, Poçan et al. (2023) found that the use of mobile technology, augmented reality applications, and online communication groups in algebra instruction was associated with better achievement and motivation outcomes in the experimental group than in the control group. However, differences were not found across all motivational subdimensions, particularly intrinsic goal orientation and test anxiety. These findings indicate that technology can support learning activities when integrated into appropriate instructional designs, but they do not demonstrate that individual digital competence

automatically improves mathematics readiness. Beyond individual abilities, the utilization of digital resources depends on device access, connectivity, and learning opportunities. Therefore, the social and educational context needs to be understood as an external condition shaping access to, development of, and utilization of resources, rather than as a fifth family of individual resources.

The relationship between individual resources, learning demands, and social context indicates that mathematics learning readiness cannot be reduced to the sum of learners' characteristics. The same resource profile may be adequate for one learning objective but insufficient for another that requires different prerequisites, strategies, or forms of reasoning. Relevant prior knowledge can facilitate subsequent learning, motivation can sustain engagement, self-regulation can help manage difficulties, and technology can expand access to information or representations. However, these functions do not automatically substitute for one another or guarantee mastery of the targeted competence. Empirical evidence from Alreshidi (2023), Trias et al. (2021), Yang et al. (2024), and Poçan et al. (2023) demonstrates that each resource has a distinct relationship with learning processes and outcomes and operates under particular conditions. Therefore, readiness must be determined in relation to the adequacy of resources for competence goals, task demands, solution pathways, forms of assistance, and contextual conditions. This relationship provides the foundation for developing the *Dynamic Mathematics Learning Readiness* (D-MLR) framework.

Development of the Dynamic Mathematics Learning Readiness (D-MLR) framework

The Dynamic Mathematics Learning Readiness (D-MLR) framework defines mathematics learning readiness as the prospective adequacy of learners' resources to engage in activities directed toward specific competence goals. This adequacy is determined by the relationship between available resources and the conditions of mathematical activities, rather than by learners' characteristics considered in isolation. Accordingly, the unit of analysis in D-MLR is the learner–task–time relationship. This perspective is consistent with Schoenfeld (2010), who identifies goals and resources as important elements in mathematical decision-making. However, D-MLR specifically uses this relationship to anticipate readiness before an activity takes place. Readiness is not inferred from successful task completion because the target competence does not necessarily need to be mastered before learners are given the opportunity to develop it.

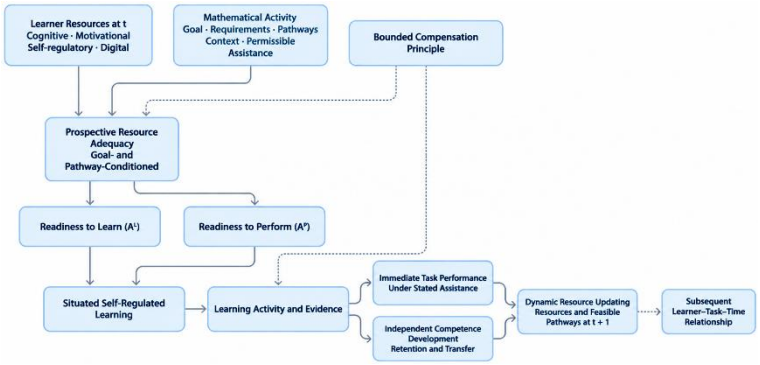

Figure 1. Conceptual Relationships in the Dynamic Mathematics Learning Readiness (D-MLR) Framework.

Source: Developed by the author.

Figure 1 illustrates the conceptual structure of D-MLR through the relationship between learner resources and the conditions of mathematical activities. Resources at time (t) comprise cognitive, motivational, self-regulatory, and digital dimensions, whereas activity conditions include goals, task demands, solution pathways, context, and permissible assistance. Their interaction produces prospective resource adequacy, which gives rise to two dimensions of readiness: readiness to learn ((A^L)) and readiness to perform ((A^P)). These forms of readiness are associated with self-regulation in learning situations, which subsequently leads to immediate task performance and independent competence development, including retention and transfer. Learning outcomes provide the basis for updating resources and solution pathways at the subsequent time point. The bounded compensation principle governs how assistance is accounted for and how task outcomes are interpreted. The relationships depicted in the diagram represent conceptual propositions rather than empirically validated causal pathways.

D-MLR uses two groups of inputs: learner resources and the conditions of mathematical activities. Resources at time (t) are expressed as $R_i(t) = (C_i, M_i, S_i, D_i)$, encompassing cognitive, motivational, self-regulatory, and digital dimensions. These four dimensions are proposed analytical categories rather than empirically validated latent factors. This distinction is important because motivation is associated with success expectations and activity value (Eccles & Wigfield, 2020), whereas self-regulation involves interactions between individual characteristics and task demands (Efklides, 2011). In a study involving 279 eighth- and ninth-grade students, Holenstein et al. (2022) found that self-efficacy predicted mathematical modeling ability after prior achievement and reasoning were taken into account. This finding supports the relevance of motivational resources, not their role as substitutes for mathematical knowledge. Digital competence also encompasses the ability to use and evaluate technology, rather than merely owning devices (Vuorikari et al., 2022). Context,

meanwhile, represents an external condition that influences learning opportunities, not a fifth dimension of internal resources.

The interaction between the two input groups produces *prospective resource adequacy*, which refers to the assessment of whether available resources are sufficient to enter or complete an activity in relation to a particular goal and pathway. For task (j) with goal (g), the set of admissible pathways is expressed as $P_j(g) = \{p_1, \dots, p_m\}$. Each pathway has completion attributes Q_{jp}, activity prerequisites $E_{jpg} \subseteq Q_{jp}$, and target reasoning attributes $T_{jpg} \subseteq Q_{jp}$. This distinction prevents learning prerequisites from being conflated with the competence being developed. For example, understanding the distributive property may serve as a prerequisite for learning factorization, but it does not demonstrate mastery of factorization. Cognitive diagnostic models, including G-DINA and multistrategy diagnosis, provide a foundation for mapping attributes and variations in solution pathways (de la Torre, 2011; Ma & Guo, 2019). Rittle-Johnson and Star (2007) also demonstrate the importance of considering solution methods rather than merely final answers. Accordingly, adequacy must be assessed in relation to the goals and requirements of each pathway, rather than through general scores alone.

Based on prospective resource adequacy, D-MLR distinguishes between *readiness to learn* A^L and *readiness to perform* A^P. These represent two distinct prospective decisions: readiness to enter a learning activity and readiness to complete a task under specified conditions. Let U_i denote the attributes independently demonstrated by a learner, $B_{jp}(h)$ the attributes supplied through assistance (h), and $O_{jpg}(h)$ the target reasoning attributes that learners are still required to generate themselves. The indicators are formulated as follows: $A^L_{ijg}(t,h) = \max_{p \in P_j(g)} \{b_{ijp}(h) \times I[E_{jpg} \subseteq U_i \cup B_{jp}(h)] \times I[T_{jpg} \subseteq O_{jpg}(h)]\}$. The function $I(\cdot)$ takes a value of one when the specified condition is satisfied. Thus, A^L may equal one while A^P equals zero, for example, when learners possess the prerequisites for learning a procedure but cannot yet complete it independently. Conversely, assistance may enable A^P to equal one. This formulation constitutes a conceptual decision rule, not a probability or a validated measure.

The *bounded compensation principle* governs how external assistance is accounted for in determining readiness and interpreting evidence of learning. Assistance may compensate for missing prerequisites to enable learners to enter an activity, but reasoning supplied by external sources does not automatically constitute evidence of independent competence. For example, a prompt to initiate factorization may support the solution of a quadratic equation. However, if an external system provides the entire factorization process and determines the roots, the resulting answer does not demonstrate mastery of the underlying reasoning. This principle is related to the *assistance dilemma*, which concerns determining what information should be provided to ensure that assistance continues to support learning (Koedinger & Aleven, 2007),

and the *expertise reversal effect*, which indicates that the appropriateness of assistance depends on learners' prior knowledge (Kalyuga et al., 2003). In a randomized experiment involving 274 participants, Pardos and Bhandari (2024) found that ChatGPT assistance improved learning outcomes compared with no assistance, with gains that did not differ significantly from those achieved through human tutoring. However, 32% of ChatGPT-generated assistance failed quality checks before mitigation. These findings demonstrate the potential benefits of assistance as well as the importance of examining its accuracy and function. Therefore, assistance must be evaluated in relation to competence goals and the reasoning that learners are still expected to perform themselves.

Once readiness has been determined, learning involves *situated self-regulated learning*, referring to the mobilization of resources during specific mathematical activities. Learners may select strategies, monitor their solutions, evaluate procedures, modify their approaches, or decide when assistance is needed. Therefore, the availability of self-regulatory resources is not equivalent to their actual use during an activity. Efklides (2011) explains the interaction of metacognition, motivation, and affect at both the individual level and the level of task engagement. Winne and Hadwin (1998) describe regulation as a process involving task understanding, goal setting, strategy implementation, and adaptation based on evaluation. Panadero (2017) also demonstrates that *self-regulated learning* models encompass cognitive, metacognitive, motivational, behavioral, and affective dimensions. Empirically, through a two-wave longitudinal study involving 327 high school students, Tang et al. (2021) found a positive relationship between autonomous motivation and metacognitive strategy knowledge, which contributed to competence. Through an analysis of digital behavior involving 29 students, Hershkovitz et al. (2024) found that failure to act on feedback was associated with lower task completion success. This indicates that the availability of assistance does not guarantee its effective utilization. Within D-MLR, self-regulation explains how resources are mobilized during activities rather than merely whether such capacities are available. This relationship represents a conceptual integration, not an empirically established causal mechanism.

Learning activities produce two distinct outcomes: *immediate task performance under stated assistance* and *independent competence development*. Immediate performance ((Y)) represents task outcomes achieved with the assistance available, whereas independent competence refers to the ability to generate or explain reasoning without assistance that substitutes for the target reasoning. Changes in competence are expressed as $\Delta K_i = K_i(t+1) - K_i(t)$, provided that measurements at both time points are comparable. This distinction is consistent with Soderstrom and Bjork (2015), who demonstrate that performance during learning does not always reflect long-term learning. Accordingly, independent competence needs to be examined through retention and transfer rather than practice success alone. Changes in competence

provide the basis for updating resources and solution pathways at the subsequent time point. This idea is related to *hypothetical learning trajectories* (Simon & Tzur, 2004), adaptive expertise (Verschaffel et al., 2009), and longitudinal attribute diagnosis (Zhan & He, 2021). Through a quasi-experimental study involving 4,442 sixth-grade students in Taiwan, Lin et al. (2026) found improvements in self-regulation and mathematics achievement associated with the use of an adaptive platform, with self-regulation partially mediating this relationship. These findings support the possibility that resources change through learning experiences, but they do not directly test the four dimensions of D-MLR or transitions between solution pathways. Thus, resource updating is a dynamic process that is not necessarily positive or uniform; the overall relationships still require empirical investigation.

Conclusion

The Dynamic Mathematics Learning Readiness (D-MLR) framework advances a new conceptualization of mathematics learning readiness as the prospective adequacy of learner resources in relation to specific goals, demands, and task solution pathways, rather than as a fixed learner characteristic. This conceptualization is important because readiness to learn a particular competence is not equivalent to the ability to complete a task independently from the outset. D-MLR explains this distinction through two dimensions: *readiness to learn* A^L and *readiness to perform* A^P, while accounting for cognitive, motivational, self-regulatory, and digital resources. Its decision-making mechanism operates by matching learner resources with the prerequisites and reasoning demands of each solution pathway, while also limiting the extent to which external assistance can compensate for missing resources. Furthermore, self-regulation explains how resources are mobilized during learning, while distinguishing immediate performance from independent competence development makes it possible to understand readiness as a dynamic process that is updated through learning experiences.

The main contribution of D-MLR lies in providing an analytical framework for distinguishing learning readiness, readiness to perform a task, and independent competence in mathematics learning. The framework also offers a conceptual basis for designing diagnostic assessments and learning support that are more responsive to competence goals, solution pathways, and the forms of assistance employed. However, D-MLR remains a conceptual framework; its four resource dimensions, mathematical decision rules, and relationships among components have not yet been empirically tested or validated. The proposed formulation also does not provide universal adequacy thresholds or standardized instruments for measuring each dimension. Future research needs to operationalize D-MLR indicators, examine the validity and consistency of its decision rules, and evaluate its ability to distinguish readiness to learn from readiness to perform across different educational levels and

mathematical topics. Longitudinal investigations are also needed to determine how assistance, self-regulation, and learning experiences contribute to changes in resources, retention, and competence transfer. Accordingly, empirical development is an essential next step in establishing the practical utility and boundaries of D-MLR in mathematics education.

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