EDUVISTA
Journal of Education and Innovation

Volume 1, Issue 1, January–June 2026

Transforming mathematical representations: A process-based analysis of junior high school students' problem solving

Prasita Manginar Tesalonika Napitupulu[1]*, Friska Ledina Situngkir[2]

[1 2] Universitas Katolik Santo Thomas, Medan, Indonesia

*Corresponding Author: prasitanapitupulu@gmail.com

Submission: 18 Juni 2026 Revision: 1 July 2026 Accepted: 5 July 2026

Abstract

Mathematical representation is an essential competency in mathematics learning because it enables students to transform information into visual, symbolic, and verbal forms throughout the problem-solving process. However, previous studies have primarily focused on assessing the achievement of representation indicators, providing limited insight into how representation transformation occurs and at which stage errors begin to emerge. This study aims to analyze junior high school students' mathematical representation transformation during problem solving on circle topics. A descriptive qualitative approach was employed involving three eighth-grade students purposively selected from high, medium, and low representation ability categories based on the results of a mathematical representation test. Data were collected through a four-item mathematical representation test and students' written work, then analyzed descriptively. The findings revealed that the greatest difficulty occurred in transforming *visual representation* into *symbolic representation*, which subsequently affected *verbal representation* and *problem verification*. This study proposes a mathematical representation transformation model consisting of *problem comprehension, visual representation, symbolic representation, verbal representation,* and *problem verification*, providing a conceptual contribution to understanding students' mathematical thinking processes.

Keywords: Mathematical representation; Problem solving; Representation transformation; Thinking process; Junior high school students.

Abstrak

Kemampuan representasi matematis merupakan kompetensi penting dalam pembelajaran matematika karena membantu siswa mentransformasikan informasi ke dalam bentuk visual, simbolik, dan verbal selama proses penyelesaian masalah. Namun, sebagian besar penelitian terdahulu lebih berfokus pada pengukuran ketercapaian indikator representasi sehingga belum banyak mengungkap bagaimana proses transformasi representasi berlangsung dan pada tahapan mana kesalahan mulai muncul. Penelitian ini bertujuan menganalisis transformasi representasi matematis siswa SMP dalam proses penyelesaian masalah pada materi lingkaran. Penelitian menggunakan pendekatan kualitatif deskriptif dengan melibatkan tiga siswa kelas VIII yang dipilih secara *purposive* dari kategori kemampuan tinggi, sedang, dan rendah berdasarkan hasil tes kemampuan representasi matematis. Data diperoleh melalui tes tertulis dan dokumentasi hasil pekerjaan siswa, kemudian dianalisis secara deskriptif. Hasil penelitian menunjukkan bahwa hambatan terbesar terjadi pada transformasi *visual representation* menuju *symbolic representation*, yang selanjutnya memengaruhi *verbal representation* dan *problem verification*. Penelitian ini menghasilkan model transformasi representasi matematis yang terdiri atas *problem comprehension, visual representation, symbolic representation, verbal representation,* dan *problem verification* sebagai kontribusi konseptual dalam menjelaskan proses berpikir matematis siswa.

Kata Kunci: Kemampuan representasi matematis; Penyelesaian masalah; Transformasi representasi; Proses berpikir; Siswa SMP.

Introduction

Mathematical representation refers to students' ability to express, process, and communicate mathematical ideas through various forms of representation, including diagrams, symbols, tables, graphs, and verbal explanations (Goldin, 2020). This ability plays a fundamental role in mathematics education because mathematical concepts are inherently abstract and therefore require representational tools to facilitate understanding and establish connections with real-world situations (Yuanita et al., 2018). In mathematics learning, representations function not only as the final products of problem solving but also as cognitive tools that enable students to identify relevant information, construct mathematical models, manipulate symbolic expressions, and interpret the resulting solutions (Verschaffel et al., 2020). Consequently, mathematical representation serves as a foundation for developing problem-solving, reasoning, communication, and mathematical connection skills, all of which are central objectives of mathematics education at the junior high school level.

Mathematical representation is particularly important in learning circle geometry, where students are required to move flexibly among visual, symbolic, and verbal representations (Rahmah et al., 2019). When solving problems involving the area of shaded regions, arc length, sectors, and the relationships among the elements of a circle, students need more than memorizing formulas. They must be able to interpret diagrams, identify geometric relationships, construct appropriate mathematical models, and justify each step of their solutions (Battista et al., 2018). If one form of representation is not well developed, errors made at the initial stage are likely to propagate throughout the subsequent stages of problem solving, ultimately leading to incorrect solutions. This condition indicates that successful mathematical problem solving depends largely on students' ability to construct and coordinate multiple forms of representation rather than merely obtaining the correct answer.

Mathematical representation can essentially be viewed as a process of information transformation that unfolds progressively throughout problem solving (Kaur, 2019). This process begins with understanding the given information, transforming it into visual or mathematical models, manipulating symbolic expressions based on relevant concepts, and finally interpreting the obtained results through logical explanations. Consequently, representational errors are not always caused by an inability to apply mathematical formulas. Instead, they may arise when students misunderstand the problem context, construct inappropriate mathematical models, or fail to establish meaningful connections between different forms of representation. Therefore, analyzing students' solution processes is essential because it reveals how representations develop throughout their thinking processes while identifying the sources of errors that cannot be adequately explained solely through the evaluation of final answers.

Researchers have investigated mathematical representation from various perspectives. Rahma and Nur (2023) found that students experienced difficulties in visual and symbolic representations; however, their study focused primarily on the achievement of representation indicators without explaining how representational errors emerged during the problem-solving process. Azkiah and Sundayana (2022) reported that students encountered difficulties in transforming verbal representations into mathematical models, yet they did not examine the relationships among different forms of representation across the stages of mathematical thinking. Azzahra and Sopiany (2023), Prihartini and Sopiany (2023), and Hilmi (2023) emphasized the influence of mathematical disposition, self-efficacy, and *habits of mind* on students' representational abilities. Meanwhile, Puspitasari and Susanah (2022), Iffa et al. (2022), and Verina and Darhim (2023) primarily compared mathematical representations based on gender, ability level, or specific mathematical topics. Although these studies provide valuable insights, they have yet to explain comprehensively how visual, symbolic, and verbal representations are constructed and interact throughout the mathematical problem-solving process.

Based on these considerations, investigating students' solution processes in relation to mathematical representation on circle topics is both relevant and necessary. This study aims to describe how junior high school students construct visual, symbolic, and verbal representations throughout each stage of problem solving, beginning with problem comprehension, followed by planning, executing the solution, and concluding with problem verification. This focus is expected to provide a deeper understanding of the underlying sources of representational errors that have received limited attention in previous studies. The novelty of this research lies in conceptualizing mathematical representation as a transformation process across multiple forms of representation during problem solving rather than merely as an outcome measured through representation achievement indicators. The findings are expected to provide meaningful implications for mathematics teachers in designing learning experiences that emphasize students' thinking processes, thereby fostering the development of mathematical representation skills more effectively and systematically.

Method

This study employed a qualitative descriptive approach to examine students' solution processes in mathematical representation on the topic of circles (Creswell, 2014). The study was conducted during the second semester of the 2025/2026 academic year at a junior high school in Medan, Indonesia. The participants consisted of 36 eighth-grade students who completed a mathematical representation test. Subsequently, the research subjects were selected using purposive sampling based on the test results and recommendations from the mathematics teacher. The students were classified into high, medium, and low representation ability categories according to their test scores. One student from each category was then selected as the primary research subject based on the clarity and completeness of his or her written solution process. Consequently, three students were chosen for in-depth analysis because they were considered to represent the characteristics of each ability category.

The research instruments consisted of the researcher as the primary instrument, a mathematical representation test comprising four open-ended questions developed based on visual, symbolic, and verbal representation indicators on the topic of circles, and documentation of students' written work. All instruments were validated by two mathematics education experts to evaluate content validity, construct validity, and language appropriateness before being revised according to the validators' recommendations. Data collection was conducted in two stages. First, the mathematical representation test was administered to all eighth-grade students. Second, three students representing the high, medium, and low ability categories were selected based on the test results. Their written responses were subsequently analyzed in depth to identify the process of mathematical representation transformation across each stage of problem solving.

Data were analyzed using the interactive model proposed by Miles, Huberman, and Saldaña (2019), which consists of data condensation, data display, and conclusion drawing and verification. During the data condensation stage, students' written responses were coded according to the stages of problem solving, namely *problem comprehension, planning, solution execution,* and *problem verification*. Each stage was then analyzed to identify the emergence of visual, symbolic, and verbal representations throughout the representation transformation process. The findings were subsequently presented in the form of narrative descriptions supported by students' written work to illustrate how mathematical representations were constructed during problem solving. To ensure the trustworthiness of the findings, the analysis was conducted systematically by comparing the consistency of representation patterns across the written responses of the three research subjects before drawing the final conclusions.

Results and Discussion

Characteristics of Students' Solution Processes in Mathematical Representation

Students' mathematical representation ability on the topic of circles remained relatively low. Of the 36 students who completed the mathematical representation test, 25 students (70%) were classified into the very low and low categories, while only 11 students (30%) achieved the moderate category. No students reached either the high or very high category. The distribution of students' mathematical representation ability is presented in Table 1. These findings indicate that most students had not yet developed the ability to use mathematical representations effectively when solving problems. Their difficulties were not limited to the application of mathematical formulas but also emerged during the processes of interpreting information, constructing mathematical models, and explaining their solutions. Therefore, the low level of mathematical representation ability reflects obstacles in students' thinking processes across multiple stages of problem solving.

Table 1. Distribution of Eighth-Grade Students' Mathematical Representation Ability

No.	**Category**	**Number of Students**	**Percentage (%)**
1	Very Low (<30)	10	28
2	Low (30–55)	15	42
3	Moderate (55–70)	11	30
4	High (70–85)	0	0
5	Very High (>85)	0	0
Total		36	100

Source: Processed research data (2026).

One of the most common error patterns observed was students' inability to transform visual information into appropriate mathematical models. Most students were able to identify essential information and sketch the given situation, yet they failed to recognize the geometric relationships embedded in the diagram. This phenomenon is illustrated in Figure 1, where a student correctly identified the radii of the inner and outer circles and drew two concentric circles but incorrectly added the areas of both circles, obtaining a result of 468.28 cm². This error suggests that the diagram functioned merely as an illustration rather than as a basis for constructing a mathematical model. Consequently, the visual representation failed to generate an accurate symbolic representation, leading to an incorrect solution for the annular region.

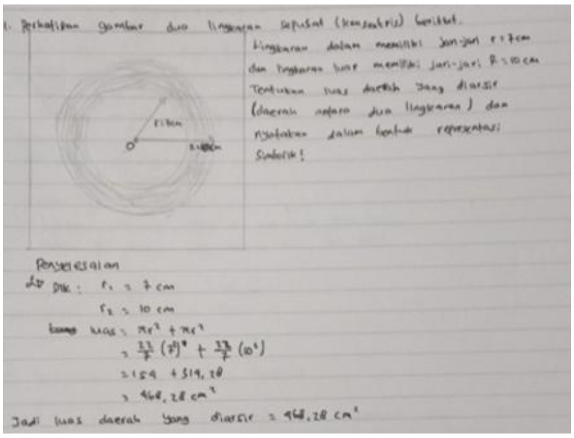
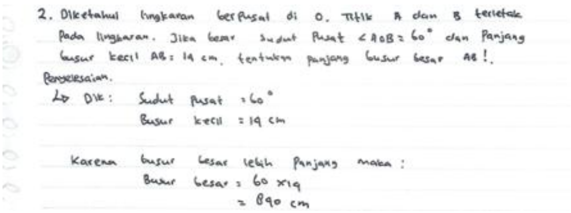


Figure 1. Student's Response to Problem 1

Source: Documentation of the research participant's written work (2026).

A similar difficulty was observed when students encountered problems requiring an understanding of relationships among mathematical quantities. Most students correctly identified the given central angle and arc length but failed to recognize how these quantities were proportionally related when determining the unknown arc length. As shown in Figure 2, one student multiplied the known arc length by the central angle, resulting in an incorrect answer of 840 cm. This procedure indicates that the student manipulated numerical values mechanically without understanding the proportional relationship between arc length and the corresponding central angle. These findings suggest that the symbolic representation constructed by the student was not grounded in conceptual understanding but rather in procedural computation. Consequently, the student was unable to provide a mathematical justification for the selected solution strategy.


2. Diketahui lingkaran berpusat di O. Titik A dan B terletak pada lingkaran. Jika besar sudut pusat ∠AOB = 60° dan panjang busur kecil AB = 14 cm, tentukan panjang busur besar AB!.

Penyelesaian.

Dik: Sudut pusat = 60°

Busur kecil = 14 cm

Karena busur besar lebih panjang maka:

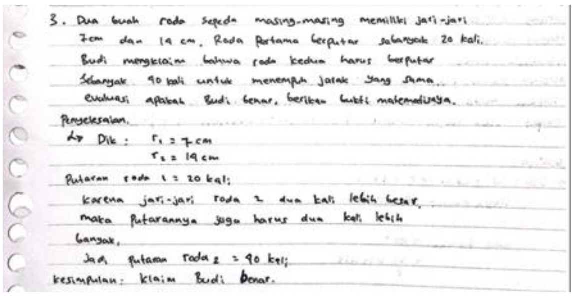
Busur besar = 60 × 14

= 840 cm

Figure 2. Student's Response to Problem 2

Source: Documentation of the research participant's written work (2026).

Representational errors were not always reflected in incorrect final answers. In several cases, students arrived at the correct conclusion but failed to construct mathematical arguments that adequately justified their reasoning. This pattern is illustrated in Figure 3, where a student correctly concluded that a larger wheel requires fewer rotations to travel the same distance. Although the conclusion was mathematically correct, the student did not demonstrate the reasoning process by relating the circumference of the wheel to the travel distance and the number of rotations. Instead, the explanation relied primarily on intuition rather than mathematical reasoning. This finding suggests that students' verbal representation was relatively stronger than their symbolic representation; however, it was not supported by a systematic mathematical model, resulting in explanations that lacked rigorous mathematical justification.


3. Dua buah roda sepeda masing-masing memiliki jari-jari 7 cm dan 14 cm. Roda pertama berputar sebanyak 20 kali. Budi mengklaim bahwa roda kedua harus berputar sebanyak 40 kali untuk menempuh jarak yang sama. Evaluasi apakah Budi benar, berikan bukti matematisnya.

Penyelesaian.

Dik: r_1 = 7 cm

r_2 = 14 cm

Putaran roda 1 = 20 kali

karena jari-jari roda 2 dua kali lebih besar, maka putarannya juga harus dua kali lebih banyak,

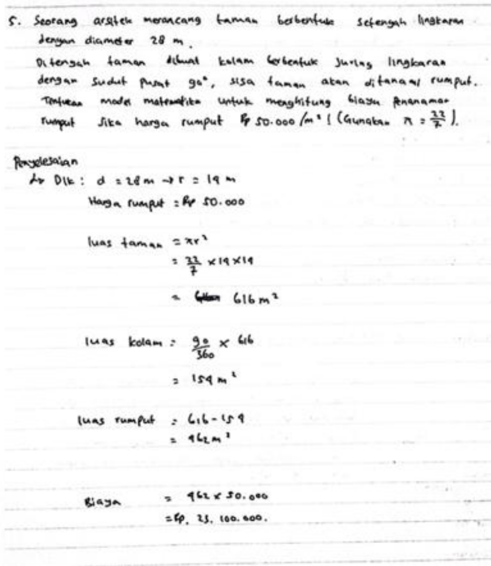
Jadi putaran roda 2 = 40 kali

kesimpulan: klaim Budi benar.

Figure 3. Student's Response to Problem 3

Source: Documentation of the research participant's written work (2026).

Another recurring error pattern revealed that although students had mastered computational procedures, they still experienced difficulties in interpreting the geometric forms presented in the problems. This situation is illustrated in Figure 4, where a student applied the formula for the area of a full circle to calculate the area of a garden that was actually semicircular. Consequently, the student calculated the garden area as 616 m², the grass-covered area as 462 m², and the planting cost as IDR 23,100,000, even though each computational step was carried out correctly. These findings indicate that the error did not stem from computational weaknesses but from an incorrect interpretation of the visual representation and its translation into an appropriate mathematical model. In other words, correct symbolic procedures do not necessarily produce correct solutions when the underlying mathematical model has been constructed inaccurately from the outset.


5. Seorang arsitek merancang taman berbentuk setengah lingkaran dengan diameter 28 m.
Ditengah taman dibuat kolam berbentuk juring lingkaran dengan sudut pusat 90°, sisa taman akan ditanami rumput.
Tentukan model matematika untuk menghitung biaya penanaman rumput jika harga rumput Rp 50.000/m² ! (Gunakan $\pi = \frac{22}{7}$).

Penyelesaian
Dik : d = 28 m → r = 14 m
Harga rumput = Rp 50.000

luas taman = πr^2
= $\frac{22}{7} \times 14 \times 14$
= 616 m²

luas kolam = $\frac{90}{360} \times 616$
= 154 m²

luas rumput = 616 − 154
= 462 m²

Biaya = 462 × 50.000
= Rp. 23. 100. 000.

Figure 4. Student's Response to Problem 4

Source: Documentation of the research participant's written work (2026).

The error patterns identified across the four problems indicate that students' mathematical representation ability had not yet developed in a balanced manner. A summary of students' performance on each representation indicator is presented in Table 2. Symbolic representation recorded the lowest achievement (20%), followed by visual representation (23%), while verbal representation achieved the highest percentage (26%). Nevertheless, all three indicators remained within the low category, with an overall average of only 23%. These findings indicate that students still encountered considerable difficulties in coordinating multiple forms of representation throughout the problem-solving process. Errors arising in one form of representation tended to influence subsequent representations, resulting in incomplete solution processes and answers that were inconsistent with the underlying mathematical concepts.

Table 2. Summary of Students' Mathematical Representation Achievement

No.	Representation Indicator	Percentage (%)	Category

1	Visual Representation	23	Low
2	Symbolic Representation	20	Very Low
3	Verbal Representation	26	Low
Overall Average		23	Low

Source: Processed research data (2026).

The findings demonstrate that weaknesses in mathematical representation should not be viewed as isolated errors but rather as interconnected processes. Students who struggled to interpret visual information were more likely to construct inaccurate symbolic models and subsequently encountered difficulties in providing logical verbal explanations for their solutions. This pattern was consistently observed across all four problems, including determining the area of an annulus, calculating arc length, evaluating claims about wheel rotations, and modeling the cost of planting grass. These results suggest that visual, symbolic, and verbal representations constitute an integrated sequence of cognitive processes rather than independent skills. Consequently, improving mathematical representation requires more than repeated practice in applying formulas. It also demands instructional approaches that systematically develop students' abilities to interpret visual information, construct appropriate mathematical models, and communicate mathematical reasoning in a logical and coherent manner.

Transformation of Students' Mathematical Representations in the Problem-Solving Process

Mathematical representation transformation describes how students convert the information presented in a problem into various forms of representation to produce a solution (Montenegro et al., 2018). Based on the overall findings, this transformation occurs progressively through problem comprehension, the construction of visual representations, the development of symbolic representations, the communication of verbal representations, and finally problem verification. These stages are interconnected, meaning that an error occurring at one stage is likely to influence the subsequent stages. This transformation pattern served as the basis for developing the conceptual model presented in Figure 5, while the empirical evidence was drawn from the distribution of students' mathematical representation ability (Table 1), the achievement of each representation indicator (Table 2), and the analysis of students' solution processes shown in Figures 1–4. Consequently, this discussion extends beyond identifying correct and incorrect answers by explaining how representational errors emerge throughout the problem-solving process.

The first stage of mathematical representation transformation is *problem comprehension*, which refers to students' ability to understand the given information before determining an appropriate solution strategy. As presented in Table 1, 25 out of 36 students (70%) were classified into the low and very low categories, indicating that representational difficulties had already emerged at the initial stage of problem solving. This pattern is illustrated in Figure 1, where students correctly identified the radii of the inner and outer circles as 7 cm and 10 cm, respectively, but failed to relate these values when determining the area of the annulus. Instead of recognizing the mathematical relationship between the two radii, students viewed the given information merely as numerical values to be calculated. Wineburg and McGrew (2019) similarly reported that students are generally able to identify the known and required information but often struggle to establish meaningful relationships among the given data before selecting a solution strategy. The present study extends this finding by demonstrating that this weakness constitutes the starting point of representational errors that subsequently propagate throughout the remaining stages of the transformation process.

The next stage involves transforming the understood information into a *visual representation*. Figure 1 shows that students were able to draw two concentric circles, while Figure 4 demonstrates that they successfully recognized the shape of the garden as a circle. Nevertheless, the diagrams they produced were not used to explore the geometric relationships among the elements of the figures. In the first problem, students failed to recognize that the shaded region represented the difference between two circles, whereas in the fourth problem they interpreted a semicircle as a complete circle. A similar tendency was reported by Vale et al. (2018), who observed that students often produce mathematical diagrams without employing them as tools for reasoning. The findings of the present study further reveal that this weakness does not remain confined to the visual stage but subsequently leads to the construction of inaccurate mathematical models during the symbolic representation stage.

The most critical stage identified in this study was the transformation into *symbolic representation*. As shown in Table 2, symbolic representation recorded the lowest achievement (20%), substantially lower than visual representation (23%) and verbal representation (26%). This pattern consistently appeared in Figures 1, 2, and 4. Students incorrectly added the areas of two circles instead of subtracting them, multiplied the arc length by the central angle to obtain an incorrect result of 840 cm, and applied the formula for the area of a complete circle to a semicircular figure. These errors indicate that students were unable to translate visual information into appropriate mathematical models. Ott et al. (2018) similarly argued that symbolic representation constitutes the most demanding stage because it requires students to integrate conceptual understanding with appropriate procedural knowledge. The present findings complement this perspective by demonstrating that symbolic errors are more likely to originate from unsuccessful transformations of visual representations than from deficiencies in computational skills.

The subsequent stage is *verbal representation*, during which students communicate their solutions through explanations, justifications, or mathematical conclusions. Although Table 2 indicates that verbal representation achieved the highest percentage among the three indicators (26%), the overall performance remained within the low category. Evidence from Figures 2 and 3 shows that most students were able to write the final answer but failed to explain the mathematical reasoning supporting their conclusions. In the problem involving wheel rotations, for example, students correctly concluded that a wheel with a larger diameter requires fewer rotations to travel the same distance; however, they did not justify this conclusion using the concepts of circumference, travel distance, and the number of wheel rotations. Rocha (2019) similarly found that students often formulate conclusions intuitively rather than through well-supported mathematical arguments. The present study further demonstrates that weaknesses in verbal representation are not isolated but instead result from inadequately constructed symbolic representations developed during the preceding stage.

The final stage of the transformation process is *problem verification*, which refers to students' ability to evaluate both the solution process and the obtained results. Based on the analysis of Figures 1–4, most students did not re-examine their answers after completing the problems. This was evident from the absence of any attempt to correct solutions that clearly contradicted the problem context, such as adding the areas of two circles, obtaining an arc length of 840 cm, or applying the formula for the area of a complete circle to a semicircle. These findings suggest that students were more concerned with obtaining numerical answers than with evaluating the validity of their solution processes. Salido and Dasari (2019) similarly explained that the reflection stage is frequently overlooked because students tend to consider problem solving complete once an answer has been obtained. The present findings further indicate that the absence of verification allows errors occurring in earlier stages to remain undetected, ultimately affecting the final solution.

The transformation stages identified in this study demonstrate that representational errors do not occur independently but rather form an interconnected sequence throughout the problem-solving process. Difficulties encountered during *problem comprehension* influence the quality of *visual representation*, which subsequently affects the construction of *symbolic representation*, weakens *verbal representation*, and ultimately leads to inadequate *problem verification*. This pattern was consistently observed across Figures 1–4, where errors introduced at earlier stages were systematically carried forward into subsequent stages. Synthesizing these findings, this study proposes a model of students' mathematical representation transformation, presented in Figure 5. The model illustrates that mathematical problem solving develops through five interconnected stages: *problem comprehension, visual representation, symbolic representation, verbal representation,* and *problem verification*. Unlike Hanifah et al. (2020), who emphasized the achievement of representation indicators, the present study demonstrates that the relationships among these stages play a more fundamental role in determining successful mathematical problem solving.

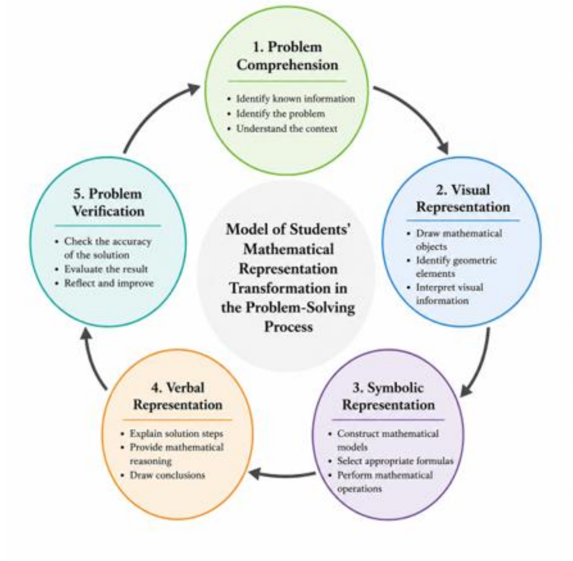


Figure 5. A Model of Students' Mathematical Representation Transformation in the Problem-Solving Process Based on the Research Findings

Source: Developed by the researchers based on the research findings (2026).

The model presented in Figure 5 illustrates that mathematical representation should be understood as a continuous transformation process rather than as a collection of independent abilities. The analyses of Tables 1 and 2 together with Figures 1–5 reveal that the greatest obstacle occurs during the transformation from *visual representation* to *symbolic representation*, as reflected by the low symbolic representation achievement of only 20%. When students fail to interpret geometric relationships accurately at the visual stage, they construct inappropriate mathematical models, which subsequently affects their verbal explanations and problem verification. Therefore, improving students' mathematical representation ability requires more than repeated practice in applying formulas or computational procedures. Instead, mathematics instruction should emphasize learning experiences that systematically develop students' ability to transform information across different forms of representation. The principal contribution of this study lies in proposing a process-oriented perspective, demonstrating that successful mathematical problem solving depends more on the quality of representation transformation than on the correctness of the final answer alone.

Conclusion

This study demonstrates that junior high school students' mathematical representation ability on the topic of circles remains relatively low, with most students classified into the low and very low categories. The primary challenge lies not merely in applying mathematical formulas but in transforming information from one form of representation into another. The findings indicate that students are generally able to identify the information presented in a problem and construct simple visual representations; however, they encounter considerable difficulties when translating these representations into appropriate mathematical models. Weaknesses at the *symbolic representation* stage subsequently affect students' ability to provide logical verbal explanations and to verify the accuracy of their solutions. These findings suggest that representational errors occur as a sequential process, indicating that successful mathematical problem solving depends more on the quality of representation transformation than on the correctness of the final answer alone.

This study contributes conceptually by proposing a model of students' mathematical representation transformation that illustrates the relationships among *problem comprehension, visual representation, symbolic representation, verbal representation,* and *problem verification* throughout the problem-solving process. The proposed model extends existing research on mathematical representation, which has primarily emphasized the achievement of representation indicators, by demonstrating that the interactions among transformation stages play a fundamental role in determining the quality of mathematical problem solving. Nevertheless, this study is limited by the small number of participants drawn from a single school and its exclusive focus on circle topics, restricting the generalizability of the findings to broader educational contexts. Future research is therefore encouraged to involve more diverse participants, investigate additional mathematical topics, and develop instructional strategies specifically designed to strengthen students' ability to transform visual, symbolic, and verbal representations in an integrated manner throughout the problem-solving process.

References

Azkiah, F., & Sundayana, R. (2022). Kemampuan representasi matematis siswa SMP berdasarkan self-efficacy siswa. *Plusminus: Jurnal Pendidikan Matematika*, *2*(2), 221–232. https://doi.org/10.31980/plusminus.v2i2.1099

Azzahra, F. P., & Sopiany, H. N. (2023). Kemampuan representasi matematis siswa berdasarkan disposisi matematis siswa SMP. *Didactical Mathematics*, *5*(1), 96–106. https://doi.org/10.31949/dm.v5i1.5048

Battista, M. T., Frazee, L. M., & Winer, M. L. (2018). Analyzing the relation between spatial and geometric reasoning for elementary and middle school students. In *Visualizing mathematics: Research in mathematics education* (pp. 195–228). Springer. https://doi.org/10.1007/978-3-319-98767-5_10

Creswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches* (4th ed.). SAGE Publications.

Goldin, G. A. (2020). Mathematical representations. In *Encyclopedia of mathematics education* (pp. 566–572). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_103

Hanifah, Waluya, S. B., Rochmad, & Wardono. (2020). Mathematical representation ability and self-efficacy. *Journal of Physics: Conference Series*, *1613*(1), Article 012062. https://doi.org/10.1088/1742-6596/1613/1/012062

Hilmi, Y. (2023). Analisis kemampuan representasi matematis siswa SMP ditinjau dari habits of mind. *J-PiMat: Jurnal Pendidikan Matematika*, *5*(2), 979–988. https://doi.org/10.31932/j-pimat.v5i2.2906

Iffa, D. S., Subarinah, S., Baidowi, B., & Sripatmi, S. (2022). Kemampuan representasi matematis siswa dalam memecahkan masalah relasi dan fungsi kelas VIII SMP. *Jurnal Riset Pendidikan Matematika Jakarta*, *4*(1), 1–11. https://doi.org/10.21009/jrpmj.v4i1.23021

Kaur, B. (2019). The why, what and how of the 'Model' method: A tool for representing and visualising relationships when solving whole number arithmetic word problems. *ZDM*, *51*(1), 151–168. https://doi.org/10.1007/s11858-018-1000-y

Miles, M. B., Huberman, A. M., & Saldaña, J. (2019). *Qualitative data analysis: A methods sourcebook* (4th ed.). SAGE Publications.

Montenegro, P., Costa, C., & Lopes, B. (2018). Transformations in the visual representation of a figural pattern. *Mathematical Thinking and Learning*, *20*(2), 91–107. https://doi.org/10.1080/10986065.2018.1441599

Ott, N., Brünken, R., Vogel, M., & Malone, S. (2018). Multiple symbolic representations: The combination of formula and text supports problem solving in the mathematical field of propositional logic. *Learning and Instruction*, *58*, 88–105. https://doi.org/10.1016/j.learninstruc.2018.04.010

Puspitasari, N. D., & Susanah, S. (2022). Analisis representasi matematis siswa SMP dalam memecahkan masalah aritmatika sosial. *MATHEdunesa*, *11*(3), 958–967. https://doi.org/10.26740/mathedunesa.v11n3.p958-967

Rahma, R. A., & Nur, I. R. D. (2023). Analisis kemampuan representasi matematis siswa SMP pada materi himpunan. *PRISMA*, *12*(1), 105. https://doi.org/10.35194/jp.v12i1.2877

Rahmah, F., Subanji, & Irawati, S. (2019). Mathematical representation analysis of students in solving mathematics problems. *Journal of Physics: Conference Series*, *1200*, Article 012011. https://doi.org/10.1088/1742-6596/1200/1/012011

Rocha, H. (2019). Mathematical proof: From mathematics to school mathematics. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and*

Engineering Sciences, *377*(2140), Article 20180045. https://doi.org/10.1098/rsta.2018.0045

Salido, A., & Dasari, D. (2019). The analysis of students' reflective thinking ability viewed by students' mathematical ability at senior high school. *Journal of Physics: Conference Series*, *1157*, Article 022121. https://doi.org/10.1088/1742-6596/1157/2/022121

Vale, I., Pimentel, T., & Barbosa, A. (2018). The power of seeing in problem solving and creativity: An issue under discussion. In *Broadening the scope of research on mathematical problem solving* (pp. 243–272). Springer. https://doi.org/10.1007/978-3-319-99861-9_11

Verina, I., & Darhim, D. (2023). Representasi matematis siswa SMP pada materi bangun ruang. *Journal on Mathematics Education Research (J-MER)*, *4*(2), 125–138. https://doi.org/10.17509/j-mer.v4i2.65077

Verschaffel, L., Schukajlow, S., Star, J., & Van Dooren, W. (2020). Word problems in mathematics education: A survey. *ZDM*, *52*(1), 1–16. https://doi.org/10.1007/s11858-020-01130-4

Wineburg, S., & McGrew, S. (2019). Lateral reading and the nature of expertise: Reading less and learning more when evaluating digital information. *Teachers College Record: The Voice of Scholarship in Education*, *121*(11), 1–40. https://doi.org/10.1177/016146811912101102

Yuanita, P., Zulnaidi, H., & Zakaria, E. (2018). The effectiveness of Realistic Mathematics Education approach: The role of mathematical representation as mediator between mathematical belief and problem solving. *PLOS ONE*, *13*(9), Article e0204847. https://doi.org/10.1371/journal.pone.0204847