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Mapping students' mathematical reasoning ability in solving systems of linear equations in two variables based on NCTM indicators

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Abstract

Mathematical reasoning is an essential competency in mathematics education because it enables students to understand concepts, construct logical arguments, and solve problems effectively. This study aimed to analyze the mathematical reasoning ability of ninth-grade students in solving SLETV problems based on the five reasoning indicators proposed by the National Council of Teachers of Mathematics (NCTM). A descriptive quantitative approach was employed involving 30 ninth-grade students of SMP Negeri 1 Baktiraja. Data were collected using essay tests and analyzed through descriptive statistics based on percentage achievement. The findings revealed that mathematical manipulation achieved the highest percentage (50.00%), whereas providing mathematical justification (26.67%) and verifying solutions (20.00%) were the weakest indicators. These results indicate that students' reasoning remains predominantly procedural rather than argumentative and reflective. The novelty of this study lies in developing a comprehensive profile of students' mathematical reasoning based on NCTM indicators and proposing two conceptual models, namely the Conceptual Hierarchy of Mathematical Reasoning Ability and the Reasoning Pathway in Solving SLETV Problems, to explain the characteristics of students' mathematical reasoning.

Keywords: Mathematical reasoning ability; Systems of linear equations in two variables; NCTM; SLETV; Mathematics education.

Abstrak

Kemampuan penalaran matematis merupakan kompetensi esensial dalam pembelajaran matematika karena mendukung siswa dalam memahami konsep, menyusun argumentasi, dan menyelesaikan masalah secara logis. Penelitian ini bertujuan menganalisis kemampuan penalaran matematis siswa kelas IX pada materi SPLDV berdasarkan lima indikator *National Council of Teachers of Mathematics* (NCTM). Penelitian menggunakan pendekatan deskriptif kuantitatif dengan subjek 30 siswa kelas IX SMP Negeri 1 Baktiraja, sedangkan data dikumpulkan melalui tes uraian dan dianalisis menggunakan statistik deskriptif berbasis persentase. Hasil penelitian menunjukkan bahwa kemampuan melakukan manipulasi matematika memperoleh persentase tertinggi (50,00%), sedangkan kemampuan memberikan alasan atau bukti (26,67%) dan memeriksa kebenaran solusi (20,00%) menjadi indikator terendah. Temuan ini menunjukkan bahwa kemampuan siswa masih didominasi aspek prosedural dibandingkan argumentatif dan reflektif. Penelitian ini berkontribusi melalui penyusunan profil kemampuan penalaran matematis berdasarkan indikator NCTM serta pengembangan model konseptual *Conceptual Hierarchy of Mathematical Reasoning Ability* dan *Reasoning Pathway in Solving SPLDV Problems* sebagai sintesis hasil penelitian.

Kata Kunci: Kemampuan penalaran matematis; Sistem persamaan linear dua variabel; NCTM; SPLDV; Pembelajaran matematika.

Introduction

Mathematics is a fundamental discipline that plays a vital role in developing students' logical, analytical, systematic, critical, and creative thinking skills (Nilimaa, 2023). One of the essential competencies that should be fostered through mathematics education is mathematical reasoning ability, as it provides the foundation for understanding concepts, constructing arguments, drawing conclusions, and solving mathematical problems logically (Hasanah et al., 2019). Mathematical reasoning is also recognized as a key component of higher-order thinking skills (HOTS) required for twenty-first-century learning. Nevertheless, Indonesian students' mathematical reasoning ability remains relatively low, causing many students to encounter difficulties when solving problems that require analysis, argumentation, proof, and logical decision-making (Sumarsih et al., 2018). This condition indicates that strengthening students' mathematical reasoning ability continues to be one of the major challenges in mathematics education.

One mathematical topic that requires well-developed reasoning skills is Systems of Linear Equations in Two Variables (SLETV). This topic requires students not only to perform algebraic manipulations but also to understand the relationships between variables, formulate mathematical models from contextual situations, determine appropriate solution strategies, justify each solution step, and verify the correctness of the obtained solutions (Gardenia et al., 2021). However, classroom practice shows that although many students can successfully solve routine SLETV problems, they often struggle with non-routine tasks requiring reasoning, justification, and interpretation of mathematical solutions (Pratiwi et al., 2021). This finding suggests that procedural proficiency has not yet been accompanied by adequate mathematical reasoning ability.

The low level of students' mathematical reasoning ability is presumably associated with instructional practices that continue to emphasize procedural problem-solving and the attainment of correct answers (Sinaga et al., 2023). Mathematics instruction frequently prioritizes formulas and algorithmic procedures over cognitive processes that encourage students to formulate conjectures, provide mathematical justification, draw conclusions, and verify the validity of their solutions (Haavold & Sriraman, 2022). Consequently, students become accustomed to following demonstrated procedures rather than independently constructing conceptual understanding and mathematical arguments. This instructional orientation ultimately limits students' ability to solve mathematical problems requiring higher-order reasoning, particularly those involving SLETV.

Previous studies on mathematical reasoning ability in SLETV have consistently reported that students' performance remains at low to moderate levels. Jankvist and Niss (2020) found that students experienced difficulties in formulating conjectures, performing mathematical manipulations, and drawing logical conclusions; however, their study was limited to describing students' levels of reasoning without identifying the types of errors associated with each reasoning indicator. Dede (2019) reported that students were unable to provide adequate mathematical justification for the procedures they applied, although the study focused exclusively on argumentative ability without examining all dimensions of mathematical reasoning. Yana and Wijaya (2025) revealed that students tended to follow algorithmic procedures without understanding the conceptual relationships underlying SLETV problems; nevertheless, their study did not explain the relationships among reasoning indicators or the characteristics of students' difficulties within each indicator. Collectively, these studies have primarily described students' levels of mathematical reasoning without providing a comprehensive profile across individual reasoning indicators or identifying the specific difficulties encountered by students. Therefore, further research is needed to address this gap and provide a more comprehensive understanding of students' mathematical reasoning ability in SLETV.

Based on these issues, the present study aims to analyze the mathematical reasoning ability of ninth-grade junior high school students in solving Systems of Linear Equations in Two Variables (SLETV) problems using the five reasoning indicators proposed by the National Council of Teachers of Mathematics (NCTM), namely formulating conjectures, performing mathematical manipulations, drawing conclusions, providing mathematical justification, and verifying solutions. In addition, this study seeks to identify the specific difficulties students encounter within each reasoning indicator in order to develop a more comprehensive profile of their mathematical reasoning ability. The findings are expected to provide empirical evidence for designing more effective instructional strategies, developing appropriate assessment instruments, and improving mathematics learning activities aimed at enhancing students' mathematical reasoning ability.

Method

This study employed a quantitative approach using a descriptive research design to examine students' mathematical reasoning ability in the topic of Systems of Linear Equations in Two Variables (SLETV) based on five mathematical reasoning indicators. The study was conducted at SMP Negeri 1 Baktiraja during the second semester of the 2025/2026 academic year (Creswell, 2014). The participants consisted of all 30 students in Class IX-E, who were selected using a total sampling technique, whereby every member of the target population was included as a research participant. A descriptive design was adopted because the study did not aim to examine the effect of an instructional intervention but rather to map students' mathematical reasoning ability based on the achievement of each reasoning indicator, thereby providing a comprehensive profile of students' mathematical reasoning characteristics in solving SLETV problems.

The research instrument consisted of an essay test developed based on the five mathematical reasoning indicators proposed by the National Council of Teachers of Mathematics (NCTM), namely formulating conjectures, performing mathematical manipulations, drawing conclusions, providing mathematical justification, and verifying solutions (Celedón-Pattichis et al., 2018). The test comprised four essay items representing four cognitive levels of Bloom's revised taxonomy: C3 (application), C4 (analysis), C5 (evaluation), and C6 (creation). The instrument was developed in accordance with the research indicators and the SLETV curriculum taught at the junior high school level. Subsequently, it was reviewed by the research supervisors and a mathematics teacher to ensure the alignment of the reasoning indicators, learning content, and item difficulty. In addition to the essay test, documentation was collected as supporting data to complement information related to the implementation of the study.

The study was conducted through several stages, including instrument development, instrument review, test administration, scoring of students' responses, data analysis, and conclusion drawing. Students' responses were evaluated using a scoring rubric developed according to the achievement criteria of each mathematical reasoning indicator. The data were analyzed using descriptive statistics by calculating the percentage achievement of each mathematical reasoning indicator using the following formula:

$$P = \frac{f}{N} \times 100\%$$

where:

- P = percentage of indicator achievement;
- f = number of students who achieved the indicator; and
- N = total number of participants.

The resulting percentages were subsequently interpreted according to predetermined achievement categories and then described to illustrate students' mathematical reasoning ability for each indicator. Furthermore, the analysis was used to identify the specific difficulties encountered by students in solving SLETV problems, thereby providing a more comprehensive understanding of the characteristics of their mathematical reasoning ability.

Results and Discussion

Achievement and Characteristics of Students' Mathematical Reasoning Ability in Systems of Linear Equations in Two Variables

This study aimed to describe the achievement and characteristics of ninth-grade students' mathematical reasoning ability in the topic of Systems of Linear Equations in Two Variables (SLETV) based on five mathematical reasoning indicators, namely formulating conjectures, performing mathematical manipulations, drawing conclusions, providing mathematical justification, and verifying solutions. Data were collected through an essay test consisting of four items representing the cognitive levels of C3 (application), C4 (analysis), C5 (evaluation), and C6 (creation). The analysis of students' responses revealed the level of achievement for each mathematical reasoning indicator, as presented in Table 1.

Table 1. Percentage of Students' Mathematical Reasoning Ability

No.	Mathematical Reasoning Indicator	Number of Students	Percentage	Category
1	Formulating conjectures	12	40.00%	Low
2	Performing mathematical manipulations	15	50.00%	Moderate
3	Drawing conclusions	10	33.33%	Low
4	Providing mathematical justification	8	26.67%	Low
5	Verifying solutions	6	20.00%	Very Low

Source: Research findings (2026).

Based on Table 1 and Figure 1, the indicator of performing mathematical manipulations achieved the highest percentage, reaching 50.00% (15 students), which falls within the moderate category. This was followed by formulating conjectures at 40.00% (12 students) and drawing conclusions at 33.33% (10 students). Meanwhile, providing mathematical justification reached only 26.67% (8 students), whereas verifying solutions recorded the lowest achievement at 20.00% (6 students). These findings indicate that students were relatively more proficient in applying solution procedures than in demonstrating mathematical reasoning involving argumentation, evaluation, and verification of the obtained solutions.

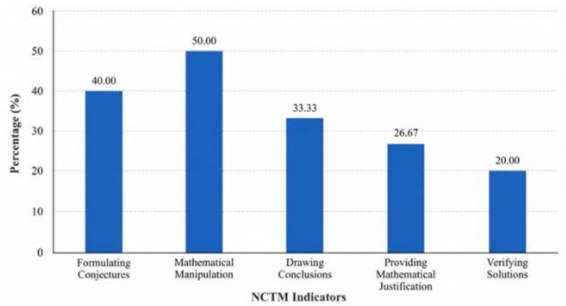
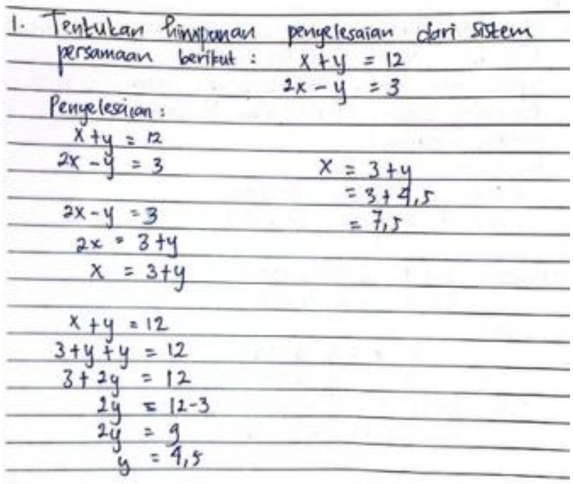


Figure 1. Percentage of Students' Mathematical Reasoning Ability Across NCTM Indicators

Source: Research findings (2026).

To obtain a more comprehensive understanding of students' mathematical reasoning characteristics, an analysis was conducted on students' responses to each test item. This analysis was intended to identify the types of errors that emerged at each cognitive level, thereby providing a more detailed description of students' mathematical reasoning ability in solving SLETV problems.



1. Tentukan himpunan penyelesaian dari sistem persamaan berikut : x + y = 12
2x − y = 3

Penyelesaian :

x + y = 12
2x − y = 3

2x − y = 3
2x = 3 + y
x = 3 + y

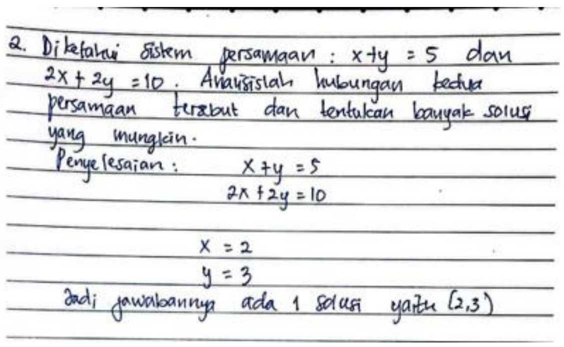
x + y = 12
3 + y + y = 12
3 + 2y = 12
2y = 12 − 3
2y = 9
y = 4,5

x = 3 + y
= 3 + 4,5
= 7,5

Figure 2. Sample of a Student's Response to the C3 (Application) Item

Source: Research documentation (2026).

As illustrated in Figure 2, the student attempted to solve the problem using the elimination and substitution methods but made errors in algebraic manipulation, resulting in an incorrect final answer. This finding is consistent with the achievement of the performing mathematical manipulations indicator, which recorded the highest percentage (50.00%), although the indicators of formulating conjectures (40.00%) and drawing conclusions (33.33%) remained in the low category. Furthermore, the student did not verify the obtained solution, explaining why verifying solutions remained the weakest indicator.



2. Diketahui sistem persamaan : x + y = 5 dan 2x + 2y = 10. Analisislah hubungan kedua persamaan tersebut dan tentukan banyak solusi yang mungkin.

Penyelesaian : x + y = 5
2x + 2y = 10

x = 2
y = 3

Jadi jawabannya ada 1 solusi yaitu (2,3)

Figure 3. Sample of a Student's Response to the C4 (Analysis) Item

Source: Research documentation (2026).

Figure 3 shows that the student successfully identified the solution pair of the system of equations but failed to analyze the relationship between the two equations as required by the problem. The student's response demonstrates a tendency to focus on obtaining the final answer without explaining the mathematical reasoning underlying the solution. This characteristic is consistent with the low achievement of the indicators of formulating conjectures, drawing conclusions, and providing mathematical justification, suggesting that students' analytical reasoning ability has not yet developed optimally.

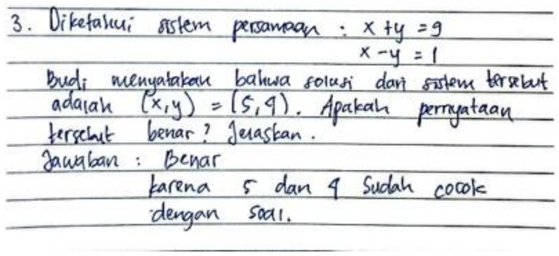

3. Diketahui sistem persamaan : x + y = 9
x − y = 1
Budi menyatakan bahwa solusi dari sistem tersebut adalah (x, y) = (5, 9). Apakah pernyataan tersebut benar? Jelaskan.
Jawaban : Benar
karena 5 dan 4 sudah cocok dengan soal.

Figure 4. Sample of a Student's Response to the C5 (Evaluation) Item

Source: Research documentation (2026).

Figure 4 illustrates that the student claimed the obtained solution was correct but did not provide any verification through substitution or mathematical justification to support the answer. This finding indicates that students are not yet accustomed to evaluating the validity of their solutions. This condition is reflected in the low achievement of providing mathematical justification (26.67%) and verifying solutions (20.00%), which were the two weakest mathematical reasoning indicators identified in this study.

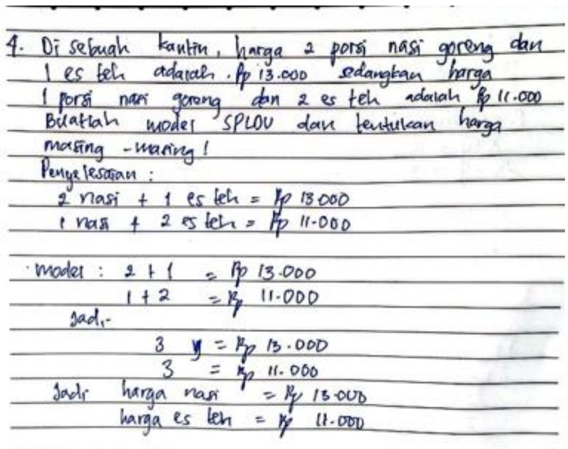

4. Di sebuah kantin, harga 2 porsi nasi goreng dan 1 es teh adalah Rp 13.000 sedangkan harga 1 porsi nasi goreng dan 2 es teh adalah Rp 11.000 Buatlah model SPLDV dan tentukan harga masing-masing!
Penyelesaian :
2 nasi + 1 es teh = Rp 13.000
1 nasi + 2 es teh = Rp 11.000
model : 2 + 1 = Rp 13.000
1 + 2 = Rp 11.000
Jadi,-
3 y = Rp 13.000
3 = Rp 11.000
Jadi harga nasi = Rp 13.000
harga es teh = Rp 11.000

Figure 5. Sample of a Student's Response to the C6 (Creation) Item

Source: Research documentation (2026).

As shown in Figure 5, the student experienced difficulties in constructing a mathematical model from a contextual problem, resulting in a solution process that did not correspond to the information provided in the problem. Errors in defining variables and constructing mathematical models contributed to students' low performance in formulating conjectures, drawing conclusions, and providing justification for the procedures employed. Moreover, students rarely verified their final solutions, indicating that their mathematical reasoning ability at the creation level remained relatively low.

Table 2. Characteristics of Students' Mathematical Reasoning Ability Across Test Items

Item	Cognitive Level	Dominant Indicator	Characteristics of Students' Responses

Item 1	C3 (Application)	Performing mathematical manipulations	Most students were able to apply elimination and substitution procedures but still made algebraic errors and rarely verified the correctness of their final answers.
Item 2	C4 (Analysis)	Formulating conjectures and drawing conclusions	Students were able to obtain solutions but were unable to analyze the relationships between equations or explain the mathematical reasoning underlying their answers.
Item 3	C5 (Evaluation)	Providing mathematical justification and verifying solutions	Most students accepted their answers as correct without verifying them through substitution, resulting in weak justification and solution verification skills.
Item 4	C6 (Creation)	Formulating conjectures and mathematical modeling	Students experienced difficulties in defining variables, constructing mathematical models, and connecting the models to logical and justifiable solution strategies.

Source: Research documentation (2026).

Based on Table 2, the characteristics of students' responses indicate that the level of difficulty increased as the cognitive demands of the test items became more complex. In the C3 item, students were relatively capable of applying SLETV solution procedures despite making several algebraic errors. In contrast, for the C4 to C6 items, most students experienced difficulties in performing analysis, providing mathematical justification, evaluating the correctness of solutions, and constructing mathematical models from contextual problems. Overall, the findings suggest that students' procedural competence was more developed than their argumentative, reflective, and evaluative reasoning abilities. These findings provide the basis for a further discussion of the factors influencing students' achievement across the mathematical reasoning indicators in solving SLETV problems.

The Dominance of Procedural Reasoning in Solving Systems of Linear Equations in Two Variables Problems

Successfully obtaining the correct answer does not necessarily indicate that students possess strong mathematical reasoning ability. In many learning situations, correct answers are often achieved because students are able to follow previously learned algorithms rather than understand the underlying mathematical concepts. This phenomenon is clearly reflected in the present study, where the indicator of performing mathematical manipulations achieved the highest percentage at 50.00% (15 students), whereas formulating conjectures reached only 40.00% (12 students) and drawing conclusions accounted for 33.33% (10 students). These differences indicate that students' reasoning ability remains predominantly procedural rather than conceptual. To illustrate this pattern more clearly, the findings were synthesized into the conceptual framework presented in Figure 6. This finding is consistent with the study by Jonsson et al. (2020), which reported that students tend to perform better on algorithmic tasks than on problems requiring conceptual reasoning. It also supports the findings of Li and Schoenfeld (2019), who argued that school mathematics instruction continues to prioritize procedural mastery over the development of mathematical thinking.

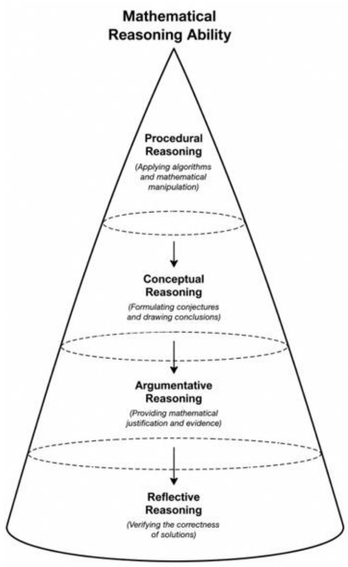


Figure 6. Conceptual Hierarchy of Students' Mathematical Reasoning Ability in Solving Systems of Linear Equations in Two Variables (SLETV) Problems

Source: Developed by the authors based on the research findings (2026).

The predominance of procedural reasoning is reflected not only in the percentage of achievement but also in the hierarchical development of students' mathematical reasoning ability. As illustrated in Figure 6, procedural reasoning occupies the highest level because it represents the reasoning ability most frequently demonstrated by students when solving SLETV problems. In contrast, conceptual reasoning, argumentative reasoning, and reflective reasoning occupy the subsequent levels because these higher-order reasoning abilities have not yet developed optimally. This hierarchy suggests that students' mathematical reasoning has not progressed in a balanced manner. Although students have become relatively proficient in executing solution procedures mechanically, they remain unable to establish conceptual relationships or critically reflect on the solution process they employ. This finding is consistent with the study by Tisngati and Genarsih (2021), which suggested that mathematical reasoning develops progressively, beginning with procedural competence before advancing toward conceptual and reflective reasoning. It also reinforces the findings of Silla et al. (2024), who identified procedural dominance as a common characteristic of algebra learning at the junior high school level.

The predominance of procedural reasoning often creates the impression that students have mastered mathematical problem solving, whereas their thinking processes remain largely mechanical and dependent on memorized procedures. This phenomenon is evident in the students' responses presented in Figure 2, where most students immediately applied the elimination and substitution methods without first analyzing the characteristics of the given system of equations. Although several students successfully followed the procedural steps, many still committed errors in algebraic manipulation, leading to incorrect final answers. This finding explains why performing mathematical manipulations recorded the highest achievement (50.00%), whereas formulating conjectures remained in the low category (40.00%). In other words, students demonstrated a stronger understanding of how to solve the problem than why a particular procedure should be applied. This observation is consistent with the findings of Demirci and Ergül (2026), who reported that repetitive practice effectively improves students' algorithmic skills.

The ability to execute mathematical procedures does not necessarily imply an understanding of the concepts underlying those procedures. This pattern is reflected in the students' responses presented in Figure 3, where students successfully determined the solution pair of the system of equations but failed to explain the equivalence relationship between the two equations as required by the task. Their responses indicate that the problem-solving process ended once the correct answer had been obtained, without any analysis of the relationship between the equations or the mathematical reasoning supporting the solution. This finding suggests that students' procedural competence has not yet evolved into comprehensive conceptual understanding. In fact, mathematical reasoning requires students not only to obtain correct answers but also to understand the logical principles underlying every solution step. This finding is in line with the study by Glasnovic Gracin (2018), which found that students solve routine problems more successfully than tasks requiring conceptual analysis. It also corroborates the findings of Szabo et al. (2020), who argued that obtaining the correct answer does not necessarily indicate a deep understanding of mathematical concepts.

The dominance of procedural reasoning indicates that SLETV instruction continues to emphasize the accuracy of final answers rather than the quality of students' mathematical thinking processes. As long as classroom instruction focuses primarily on applying elimination and substitution algorithms, students will become more accustomed to reproducing teacher-demonstrated procedures than to exploring alternative solution strategies. Consequently, the development of formulating conjectures (40.00%) and drawing conclusions (33.33%) lags behind performing mathematical manipulations (50.00%). This imbalance suggests that students' learning experiences remain heavily dominated by procedural activities, whereas opportunities to explore concepts, engage in mathematical discussions, and reflect on solution processes are still limited. This finding is consistent with the study by Österman and Bråting (2019), which reported that procedure-oriented instruction tends to produce students who are proficient in computation but less capable of developing conceptual reasoning. It also supports the findings of Lin (2023), who argued that limited opportunities for students to formulate conjectures and justify their mathematical thinking hinder the development of higher-order thinking skills.

The predominance of procedural reasoning should not be viewed entirely as a negative finding; rather, it indicates that students' mathematical reasoning is still at an early stage of development and needs to be guided toward conceptual, argumentative, and reflective reasoning. As illustrated in Figure 6, procedural reasoning serves as the initial foundation upon which students should build conceptual understanding, mathematical argumentation, and solution evaluation in order to develop comprehensive mathematical reasoning. Therefore, SLETV instruction should extend beyond training students to apply formulas and algorithms. Instead, learning activities should encourage students to formulate conjectures, justify the selection of solution methods, compare alternative strategies, and reflect critically on the correctness of their solutions. Through such learning experiences, students' existing procedural competence can become a foundation for developing more comprehensive mathematical reasoning ability.

Weaknesses in Students' Argumentative and Reflective Reasoning When Solving Systems of Linear Equations in Two Variables Problems

The ability to justify solutions and verify their correctness is often one of the most neglected aspects of mathematics instruction, despite being fundamental components of mathematical reasoning. Students who successfully obtain the correct answer are not necessarily able to defend the validity of that answer through logical argumentation and adequate verification. This phenomenon is evident in the present study, where the indicator of providing mathematical justification achieved only 26.67% (8 students), while verifying solutions recorded the lowest percentage at 20.00% (6 students). These findings indicate that most students stopped their reasoning process after obtaining the final answer without providing justification or re-examining the correctness of their solutions. To explain this mechanism conceptually, the findings were synthesized into the framework presented in Figure 7, which illustrates the reasoning pathway that students should ideally follow when solving SLETV problems. This finding is consistent with the study by Faulkner et al. (2019), which identified mathematical justification as one of the most challenging reasoning competencies for students to master. It also supports the findings of Rach and Ufer (2020), who argued that mathematics instruction continues to place greater emphasis on obtaining correct answers than on developing mathematical argumentation.

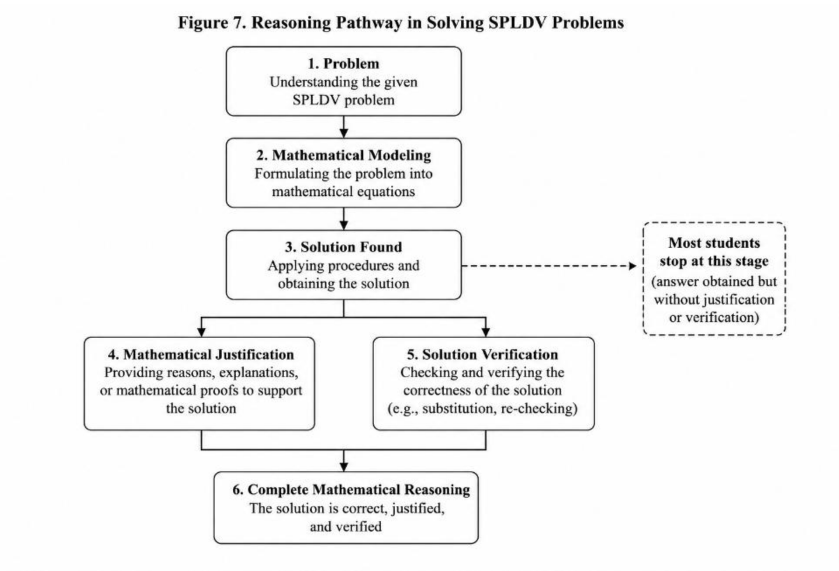


Figure 7. Reasoning Pathway in Solving Systems of Linear Equations in Two Variables (SLETV) Problems

Source: Developed by the authors based on the research findings (2026).

The weak performance in argumentative and reflective reasoning indicates that students' mathematical reasoning process remains incomplete, as it typically ends once a solution has been obtained. As illustrated in Figure 7, effective mathematical problem solving ideally involves six successive stages: understanding the problem, constructing a mathematical model, obtaining a solution, providing mathematical justification, verifying the solution, and ultimately achieving complete mathematical reasoning. However, the findings of this study reveal that most students reached only the *solution found* stage before immediately assuming that their answers were correct, without proceeding to *mathematical justification* or *solution verification*. This pattern explains why providing mathematical justification reached only 26.67%, whereas verifying solutions recorded the lowest achievement at 20.00%. This finding is consistent with the study by Veenman and van Cleef (2019), which reported that students tend to terminate their thinking process once they obtain an answer. It also corroborates the findings of Czocher (2018), who argued that mathematical reasoning can only be considered complete when solutions are accompanied by both justification and systematic verification.

The tendency to accept answers without verification suggests that students are not yet accustomed to evaluating their own reasoning processes. This pattern is evident in the students' responses presented in Figure 4, where most students immediately concluded that their answers were correct without substituting the obtained values back into the equations or providing mathematical justification to support their conclusions. Yet, verification is an essential step for ensuring that a solution satisfies the given system of equations. The low achievement of the verifying solutions indicator (20.00%) indicates that reflective thinking has not yet become an integral part of students' learning habits. Consequently, answers are accepted based on personal confidence rather than mathematical evidence. This finding is consistent with the study by Li and Yuan (2022), which reported that students rarely recheck their solutions unless explicitly instructed by their teachers. It also reinforces the findings of Rivas et al. (2022), who demonstrated that classroom cultures emphasizing final answers contribute to the slow development of reflective reasoning.

Students' difficulties in constructing mathematical models further reveal weaknesses in their ability to develop mathematical arguments supporting the solution strategies they employ. As illustrated in Figure 5, students encountered difficulties in defining variables and constructing mathematical models from contextual problems. These errors not only reflect limited understanding of the problem context but also indicate that students were unable to explain why their selected mathematical models were appropriate. Consequently, mistakes made during the initial stages of problem solving propagated throughout the entire solution process, ultimately leading to incorrect answers. These findings suggest that mathematical argumentation extends beyond explaining the final answer; it also involves justifying the selection of strategies, mathematical models, and solution procedures. This finding is consistent with the study by Niss and Højgaard (2019), which emphasized that successful mathematical modeling is strongly influenced by students' ability to construct logical arguments. It also supports the findings of Stylianides (2019), who demonstrated that weak mathematical argumentation directly reduces the quality of contextual problem solving.

The weakness of students' argumentative and reflective reasoning suggests that mathematics instruction has not yet provided sufficient opportunities for students to question, evaluate, and justify their own thinking processes. As long as classroom learning continues to focus primarily on obtaining correct answers, students are likely to perceive justification and verification as optional rather than essential components of mathematical problem solving. However, as illustrated in Figure 7, these two stages are indispensable prerequisites for achieving complete mathematical reasoning. Therefore, SLETV instruction should be designed to encourage students to justify their choice of solution methods, compare alternative strategies, verify solutions through substitution, and critically reflect on whether their answers are consistent with the problem context. Through these practices, students will not only be able to obtain correct solutions but also to demonstrate and defend the mathematical validity of those solutions. This finding is consistent with the study by Almpani et al. (2023), which recommended argumentation-based mathematics instruction as an effective approach for improving students' reasoning ability. It also reinforces the findings of Tai et al. (2018), who concluded that reflective reasoning develops only when students are consistently engaged in justification and evaluation throughout every stage of mathematical problem solving.

Conclusion

This study demonstrates that students' mathematical reasoning ability in Systems of Linear Equations in Two Variables (SLETV) remains relatively low and exhibits substantial disparities across the five reasoning indicators. Performing mathematical manipulations achieved the highest level of attainment (50.00%), whereas providing mathematical justification (26.67%) and verifying solutions (20.00%) recorded the lowest achievements. These findings indicate that students are generally more proficient in applying procedural solution strategies than in developing reasoning involving mathematical argumentation and reflective thinking. This imbalance appears to stem from instructional practices that continue to emphasize algorithmic procedures and the attainment of correct answers, leading students to focus on following prescribed solution steps rather than explaining their reasoning, evaluating alternative strategies, or verifying the correctness of their solutions. Therefore, efforts to improve students' mathematical reasoning should extend beyond procedural accuracy by fostering learning environments that consistently promote reasoning, mathematical argumentation, and reflective thinking throughout the problem-solving process.

This study contributes to the existing literature by providing a comprehensive profile of students' mathematical reasoning ability based on the five reasoning indicators proposed by the National Council of Teachers of Mathematics (NCTM). In addition, it introduces two conceptual models—the *Conceptual Hierarchy of Mathematical Reasoning Ability* and the *Reasoning Pathway in Solving SLETV Problems*—to explain the developmental structure of mathematical reasoning and the reasoning processes students employ when solving SLETV problems. Nevertheless, this study has several limitations. It involved only a single class with a relatively small number of participants, limiting the generalizability of the findings. Moreover, the descriptive quantitative design did not allow for an in-depth investigation of the factors underlying students' low mathematical reasoning ability. Future studies are therefore recommended to involve larger and more diverse samples, integrate quantitative and qualitative approaches, and develop as well as evaluate instructional models specifically designed to strengthen students' conceptual, argumentative, and reflective reasoning in mathematics learning.

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